EXERCISE – III HINTS & SOLUTIONS

Sol.1 **(a)** Equation of ellipse

$$\frac{(x-1)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1$$

$\downarrow (4, b)$

$$\frac{9}{a^2} + \frac{16}{b^2} = 1 \quad ....(1)$$

ae = 5 (given) $\Rightarrow b^2 = a^2 - 25$ ....(iii)

diving (i) & (ii) ; $a^2 = 45$, $b^2 = 20$

So equation is $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{45} = 1$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

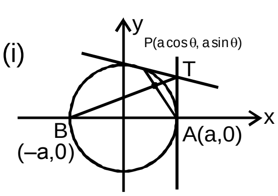
$(2, -2) \Rightarrow \frac{4}{a^2} + \frac{4}{b^2} = 1$

$(-3, 1) \Rightarrow \frac{9}{a^2} + \frac{1}{b^2} = 1$

Solve & get the value of a^2 & b^2

Sol.2 Equation of BT;

$$y = \frac{1-\cos\theta}{2\sin\theta}(x + a) \ ...(i)$$



Equation of AP :

$$y = \frac{\sin\theta}{\cos\theta - 1}(x - a) \ ...(ii)$$

Let the point of intersection is (h, k)

solve (i) & (ii) to get (h, k)

Sol.3 Equation of tangent $\frac{x\cos\alpha}{a} + \frac{y\sin\alpha}{b} = 1$ ....(i)

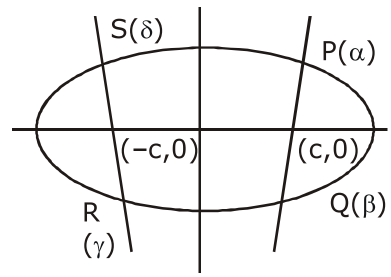
circle $x^2 + y^2 = a^2$ ....(ii)

Hogonenize the equation (ii) with the help of equation (i) & coff. of x^2 + coff. $y^2 = 0$

Sol.4 $\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$

passing (c, 0)

$$\frac{c}{a} = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$$



$$\frac{c-a}{c+a} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2} \quad ....(1)$$

$$-\frac{c}{a} = \frac{\cos\left(\frac{\gamma-\delta}{2}\right)}{\cos\left(\frac{\gamma+\delta}{2}\right)} \Rightarrow \frac{-c-a}{-c+a} = \tan\frac{\gamma}{2}\tan\frac{\delta}{2} \quad ....(2)$$

from (1) & (2)

$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}\tan\frac{\delta}{2} = 1$$

Sol.5 equation of normal at P(θ)

$ax\sec\theta - by\ \mathrm{cosec}\,\theta = a^2 - b^2$ ....(i)

$\downarrow (a\cos^2\theta, b\sin^2\theta)$ & $\cos\theta = -\frac{-2}{3}$

Sol.6 Equationi of normals at P, Q & R

$ax\sec\alpha - by\ \mathrm{cosec}\,\alpha = a^2 - b^2$ ...(i)

$ax\sec\beta - by\ \mathrm{cosec}\beta = a^2 - b^2$ ...(ii)

$ax\sec\gamma - by\ \mathrm{cosec}\,\gamma = a^2 - b^2$ ...(iii)

As the three normals arc concurrent

$$\Rightarrow \begin{vmatrix} a\sec\alpha & -b\,\mathrm{cosec}\alpha & b^2-a^2 \\ a\mathrm{sed}\beta & -b\,\mathrm{cosec}\beta & b^2-a^2 \\ a\sec\gamma & -b\,\mathrm{cosec}\gamma & b^2-a^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \sin\alpha & \cos\alpha & \sin^2\alpha \\ \sin\beta & \cos\beta & \sin^2\beta \\ \sin\gamma & \cos\gamma & \sin 2\gamma \end{vmatrix} = 0$$

Sol.7 Equation of the circle as the ends of

$$\left(ae, \frac{b^2}{a}\right) \ \& \ \left(ae, \frac{-b^2}{a}\right).$$

Sol.8 Equation of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Equation of tangents having equal ingtercepots wil have slope –1 so x + y – 5 = 0 & x + y + 5 = 0

Sol.9 $x^2 + 9y^2 - 4x + 6y + 4 = 0$

$$(x-2)^2 + \frac{(y+1/3)^2}{1/9} = 1$$

Let $x - 2 = \cos\theta \Rightarrow x = 2 + \cos\theta$

$$y + \frac{1}{3} = \frac{1}{3}\sin\theta \Rightarrow y = -\frac{1}{3} + \frac{1}{3}\sin\theta$$

$z = 4x - 9y$

$$4(2+\cos\theta) - 9\left(-\frac{1}{3} + \frac{1}{3}\sin\theta\right)$$

$= 11 + 4\cos\theta - 3\sin\theta$

$Z_{max} = 11 + 5 = 16$

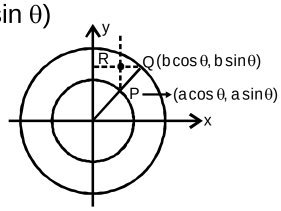
Sol.10 Equation of tangent $y = -\frac{4}{3}x + \sqrt{\frac{18.16}{9} + 32}$

$y = -\frac{4}{3}x + 8$ point P = (6, 0) & B = (0, 8)

$$\text{Area of } \Delta OAB = \left|\frac{1}{2}\begin{vmatrix} 6 & 0 & 1 \\ 0 & 8 & 1 \\ 0 & 0 & 1 \end{vmatrix}\right| = 24 \text{ sq. units}$$

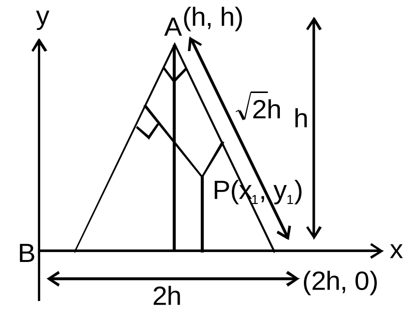
Sol.11 Point R = $(a\cos\theta, b\sin\theta)$

locus $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Sol.12 Let point P is (x_1, y_1)

Given



$$(y_1)^2 = \frac{1}{2}\left\{\left(\frac{x_1+y_1-2h}{\sqrt{2}}\right) + \left(\frac{x_1-y_1}{\sqrt{2}}\right)^2\right\}$$

Which is an ellipse.

Sol.13 Point P (x_1, y_1) tangent $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ ....(i)

$$d = \frac{-1}{\sqrt{x_1^2/a^4 + y_1^2/b^4}} \Rightarrow \frac{1}{d^2} = \frac{x_1^2}{a^4} = \frac{y_1^2}{b^4}$$

$$\text{Also } \frac{x_1^2}{a^2} + \frac{x_1^2}{a^2} = 1 \Rightarrow \frac{1}{d^2} = \frac{x_1^2}{a^4}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) + \frac{1}{b^2}$$

$$\Rightarrow \frac{b^2}{d^2} = \frac{x_1^2}{a^2}\left(\frac{b^2}{a^2} - 1\right) \Rightarrow 1 - \frac{b^2}{d^2} = \frac{e^2x_1^2}{a^2} \text{ ...(ii)}$$

Also $(PF_1 - PF_2)^2 = (a + ex_1 - a + ex_1)^2$ ...(iii)

From (ii) & (iii) $(PF_1 - PFx)^2 = 4a^2(1 - b^2/d^2)$

Sol.14 For the common tangents

$$\frac{1}{m} = \pm\sqrt{16m^2 - 6} \Rightarrow m \pm \frac{1}{2}$$

Point of intersection = (1, 0)

Find all points & then area.

Sol.15 Equation of normal at $P(\theta)$

$ax\sec\theta - by\,\mathrm{cosec}\theta = a^2 - b^2$ ...(i)

$$\text{Point G} = \left(\frac{a^2-b^2}{b}\sin\theta\right) \quad \text{...(ii)}$$

$$g = \left(0, -\frac{(a^2-b^2)}{b}\sin\theta\right) \quad \text{...(iii)}$$

& PN = b sin θ ...(iv)

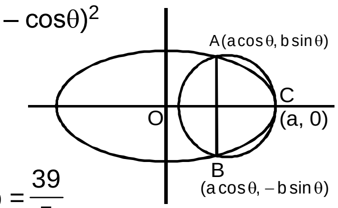
Use (ii), (iii) & (iv), (v) to get rejects.

Sol.16 AB = 2 b sin θ

AC = AB/2

$\Rightarrow b^2\sin^2\theta = a^2(1-\cos\theta)^2$

$\Rightarrow \dfrac{16}{15} = \dfrac{2\cos\theta}{1+\cos\theta}$



$\Rightarrow \sin\theta = \dfrac{15}{17}$ & $b = \dfrac{39}{5}$

so $AB = \dfrac{180}{17}$

Sol.17 Equation of tangent at P(θ)

bx cosθ + a y sinθ = ab ...(i)

point T = (a sec θ, 0)

& N = (a cos θ, 0)

Equaton of circle as NT its diameter.

(x – a sec θ) (x – a cos θ) + (y – 0) (y – 0) = 0 ...(ii)

Equation of auxillary circle

$x^2 + y^2 = a^2$ ...(iii)

for (ii) & (iii)

$2g_1g_2 + 2f_1f_2 = C_2 + C_2$

Hence they cut orthogonally

Sol.18 Let the point of contact be

(a cos θ, b sin θ) & (a cos ϕ, b sin ϕ)

as these tangents are perpendicular

So, $\tan\theta \cdot \tan\theta = -\dfrac{b^2}{a^2}$ ...(i)

as the tangents at θ & ϕ intersect at (x_1, y_1)

$\Rightarrow x_1 = \dfrac{a\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}$ & $y_1 = \dfrac{a\sin\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}$ ... (ii)

If (h, k) is point of intersection of normals at θ & ϕ then,

$h = \dfrac{a^2-b^2}{a}\ \dfrac{\cos\theta.\cos\phi.\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}$,

$k = \dfrac{a^2-b^2}{-b}\ \dfrac{\sin\phi\tan\phi\sin\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}$

Using (ii); $h = x_1\left(\dfrac{a^2-b^2}{a^2}\right)\cos\theta\,.\,\cos\phi$,

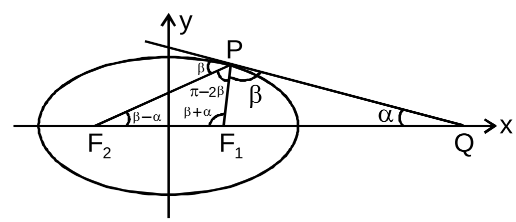
$k = y_1\left(\dfrac{a^2-b^2}{-b^2}\right)\sin\theta\sin\phi$

$\Rightarrow \dfrac{h}{k} = \left(\dfrac{-a^2}{b^2}\tan\theta\tan\phi\right) \Rightarrow \dfrac{h}{k_1} = \dfrac{y_1}{x_1}$

locus $\Rightarrow \dfrac{x}{y} = \dfrac{y_1}{x_1}$ (HP)

Sol.19 Equation of tangent at P(α)

$\dfrac{x\cos\alpha}{a} + \dfrac{y\sin\alpha}{b} = 1$ (slope = tan α)



In triangle PF_1F_2

$\dfrac{F_1P}{\sin(\beta-\alpha)} = \dfrac{F_2P}{\sin(\alpha+\beta)} = \dfrac{F_1F_2}{\sin 2\beta} = k$

and $F_1P + F_2P = 2a$

Sol.20 Center of ellipse = (29, 75/2)

foot of perpendicular from focii

lie on auxillary circle

equaton of auxillary circle

$(x-29)^2 + (y-75/2) = a^2$

↓ (9,0) foot of perpendicular

2a = 85.